RingsForHomalg

Dictionaries of external rings

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Chapter 1

Introduction

This package is part of the homalg project [hom22]. The role of the package is described in the manual of the homalg package.

1.1 Ring Constructions for Supported External Computer Algebra Systems

Here are some of the supported ring constructions:

1.1.1 external GAP

Example

```
gap> ZZ := HomalgRingOfIntegersInExternalGAP( );
Z
gap> Display( ZZ );
<An external ring residing in the CAS GAP>
gap> F2 := HomalgRingOfIntegersInExternalGAP( 2, ZZ );
GF(2)
gap> Display( F2 );
<An external ring residing in the CAS GAP>
```

`F2 := HomalgRingOfIntegersInExternalGAP( 2 )` would launch another GAP.

Example

```
gap> Z4 := HomalgRingOfIntegersInExternalGAP( 4, ZZ );
Z/4Z
gap> Display( Z4 );
<An external ring residing in the CAS GAP>
gap> Z_4 := HomalgRingOfIntegersInExternalGAP( ZZ ) / 4;
Z/( 4 )
gap> Display( Z_4 );
<A residue class ring>
gap> Q := HomalgFieldOfRationalsInExternalGAP( ZZ );
Q
gap> Display( Q );
<An external ring residing in the CAS GAP>
```

1.1.2 Singular

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Singular ).stdout )
gap> F2 := HomalgRingOfIntegersInSingular( 2 );
GF(2)
gap> Display( F2 );
<An external ring residing in the CAS Singular>
gap> F2s := HomalgRingOfIntegersInSingular( 2, "s" ,F2 );
GF(2)(s)
gap> Display( F2s );
<An external ring residing in the CAS Singular>
gap> ZZ := HomalgRingOfIntegersInSingular( F2 );
Z
gap> Display( ZZ );
<An external ring residing in the CAS Singular>
gap> Q := HomalgFieldOfRationalsInSingular( F2 );
Q
gap> Display( Q );
<An external ring residing in the CAS Singular>
gap> Qs := HomalgFieldOfRationalsInSingular( "s", F2 );
Q(s)
gap> Display( Qs );
<An external ring residing in the CAS Singular>
gap> Qi := HomalgFieldOfRationalsInSingular( "i", "i^2+1", Q );
Q[i]/(i^2+1)
gap> Display( Qi );
<An external ring residing in the CAS Singular>
#@fi
```

`Q := HomalgFieldOfRationalsInSingular( )` would launch another Singular.

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Singular ).stdout )
gap> F2xyz := F2 * "x,y,z";
GF(2)[x,y,z]
gap> Display( F2xyz );
<An external ring residing in the CAS Singular>
gap> F2sxyz := F2s * "x,y,z";
GF(2)(s)[x,y,z]
gap> Display( F2sxyz );
<An external ring residing in the CAS Singular>
gap> F2xyzw := F2xyz * "w";
GF(2)[x,y,z][w]
gap> Display( F2xyzw );
<An external ring residing in the CAS Singular>
gap> F2sxyzw := F2sxyz * "w";
GF(2)(s)[x,y,z][w]
gap> Display( F2sxyzw );
<An external ring residing in the CAS Singular>
gap> ZZxyz := ZZ * "x,y,z";
Z[x,y,z]
gap> Display( ZZxyz );
<An external ring residing in the CAS Singular>
```

```
gap> ZZxyzw := ZZxyz * "w";
Z[x,y,z][w]
gap> Display( ZZxyzw );
<An external ring residing in the CAS Singular>
gap> Qxyz := Q * "x,y,z";
Q[x,y,z]
gap> Display( Qxyz );
<An external ring residing in the CAS Singular>
gap> Qsxyz := Qs * "x,y,z";
Q(s)[x,y,z]
gap> Display( Qsxyz );
<An external ring residing in the CAS Singular>
gap> Qixyz := Qi * "x,y,z";
(Q[i]/(i^2+1))[x,y,z]
gap> Display( Qixyz );
<An external ring residing in the CAS Singular>
gap> Qxyzw := Qxyz * "w";
Q[x,y,z][w]
gap> Display( Qxyzw );
<An external ring residing in the CAS Singular>
gap> Qsxyzw := Qsxyz * "w";
Q(s)[x,y,z][w]
gap> Display( Qsxyzw );
<An external ring residing in the CAS Singular>
gap> Dxyz := RingOfDerivations( Qxyz, "Dx,Dy,Dz" );
Q[x,y,z]<Dx,Dy,Dz>
gap> Display( Dxyz );
<An external ring residing in the CAS Singular>
gap> Exyz := ExteriorRing( Qxyz, "e,f,g" );
Q{e,f,g}
gap> Display( Exyz );
<An external ring residing in the CAS Singular>
gap> Dsxyz := RingOfDerivations( Qsxyz, "Dx,Dy,Dz" );
Q(s)[x,y,z]<Dx,Dy,Dz>
gap> Display( Dsxyz );
<An external ring residing in the CAS Singular>
gap> Esxyz := ExteriorRing( Qsxyz, "e,f,g" );
Q(s){e,f,g}
gap> Display( Esxyz );
<An external ring residing in the CAS Singular>
gap> Dixyz := RingOfDerivations( Qixyz, "Dx,Dy,Dz" );
(Q[i]/(i^2+1))[x,y,z]<Dx,Dy,Dz>
gap> Display( Dixyz );
<An external ring residing in the CAS Singular>
gap> Eixyz := ExteriorRing( Qixyz, "e,f,g" );
(Q[i]/(i^2+1)){e,f,g}
gap> Display( Eixyz );
<An external ring residing in the CAS Singular>
gap> qring := HomalgQRingInSingular( Qxyz, "x*y" );
Q[x,y,z]/( x*y )
gap> Display( qring );
<An external ring residing in the CAS Singular>
```

```
gap> "z + x*y" / qring = "z" / qring;
true
#@fi
```

1.1.3 MAGMA

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_MAGMA ).stdout )
gap> ZZ := HomalgRingOfIntegersInMAGMA( );
Z
gap> Display( ZZ );
<An external ring residing in the CAS MAGMA>
gap> F2 := HomalgRingOfIntegersInMAGMA( 2, ZZ );
GF(2)
gap> Display( F2 );
<An external ring residing in the CAS MAGMA>
#@fi
```

`F2 := HomalgRingOfIntegersInMAGMA( 2 )` would launch another MAGMA.

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_MAGMA ).stdout )
gap> Z_4 := HomalgRingOfIntegersInMAGMA( ZZ ) / 4;
Z/( 4 )
gap> Display( Z_4 );
<A residue class ring>
gap> Q := HomalgFieldOfRationalsInMAGMA( ZZ );
Q
gap> Display( Q );
<An external ring residing in the CAS MAGMA>
gap> F2xyz := F2 * "x,y,z";
GF(2)[x,y,z]
gap> Display( F2xyz );
<An external ring residing in the CAS MAGMA>
gap> Qxyz := Q * "x,y,z";
Q[x,y,z]
gap> Display( Qxyz );
<An external ring residing in the CAS MAGMA>
gap> Exyz := ExteriorRing( Qxyz, "e,f,g" );
Q{e,f,g}
gap> Display( Exyz );
<An external ring residing in the CAS MAGMA>
#@fi
```

1.1.4 Macaulay2

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Macaulay2 ).stdout )
gap> ZZ := HomalgRingOfIntegersInMacaulay2( );
Z
gap> Display( ZZ );
<An external ring residing in the CAS Macaulay2>
gap> F2 := HomalgRingOfIntegersInMacaulay2( 2, ZZ );
GF(2)
```

```
gap> Display( F2 );
<An external ring residing in the CAS Macaulay2>
#@fi
```

`F2 := HomalgRingOfIntegersInMacaulay2( 2 )` would launch another Macaulay2.

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Macaulay2 ).stdout )
gap> Z_4 := HomalgRingOfIntegersInMacaulay2( ZZ ) / 4;
Z/( 4 )
gap> Display( Z_4 );
<A residue class ring>
gap> Q := HomalgFieldOfRationalsInMacaulay2( ZZ );
Q
gap> Display( Q );
<An external ring residing in the CAS Macaulay2>
gap> F2xyz := F2 * "x,y,z";
GF(2)[x,y,z]
gap> Display( F2xyz );
<An external ring residing in the CAS Macaulay2>
gap> Qxyz := Q * "x,y,z";
Q[x,y,z]
gap> Display( Qxyz );
<An external ring residing in the CAS Macaulay2>
gap> Dxyz := RingOfDerivations( Qxyz, "Dx,Dy,Dz" );
Q[x,y,z]<Dx,Dy,Dz>
gap> Display( Dxyz );
<An external ring residing in the CAS Macaulay2>
gap> Exyz := ExteriorRing( Qxyz, "e,f,g" );
Q{e,f,g}
gap> Display( Exyz );
<An external ring residing in the CAS Macaulay2>
#@fi
```

1.1.5 Sage

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Sage ).stdout )
gap> ZZ := HomalgRingOfIntegersInSage( );
Z
gap> Display( ZZ );
<An external ring residing in the CAS Sage>
gap> F2 := HomalgRingOfIntegersInSage( 2, ZZ );
GF(2)
gap> Display( F2 );
<An external ring residing in the CAS Sage>
#@fi
```

`F2 := HomalgRingOfIntegersInSage( 2 )` would launch another Sage.

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Sage ).stdout )
gap> Z_4 := HomalgRingOfIntegersInSage( ZZ ) / 4;
Z/( 4 )
```

```
gap> Display( Z_4 );
<A residue class ring>
gap> Q := HomalgFieldOfRationalsInSage( ZZ );
Q
gap> Display( Q );
<An external ring residing in the CAS Sage>
gap> F2x := F2 * "x";
GF(2)[x]
gap> Display( F2x );
<An external ring residing in the CAS Sage>
gap> Qx := Q * "x";
Q[x]
gap> Display( Qx );
<An external ring residing in the CAS Sage>
#@fi
```

1.1.6 Maple

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Maple ).stdout )
gap> ZZ := HomalgRingOfIntegersInMaple( );
Z
gap> Display( ZZ );
<An external ring residing in the CAS Maple>
gap> F2 := HomalgRingOfIntegersInMaple( 2, ZZ );
GF(2)
gap> Display( F2 );
<An external ring residing in the CAS Maple>
#@fi
```

`F2 := HomalgRingOfIntegersInMaple( 2 )` would launch another Maple.

Example

```
#@if IsBound( TryLaunchCAS_IO_ForHomalg( HOMALG_IO_Maple ).stdout )
gap> Z4 := HomalgRingOfIntegersInMaple( 4, ZZ );
Z/4Z
gap> Display( Z4 );
<An external ring residing in the CAS Maple>
gap> Z_4 := HomalgRingOfIntegersInMaple( ZZ ) / 4;
Z/( 4 )
gap> Display( Z_4 );
<A residue class ring>
gap> Q := HomalgFieldOfRationalsInMaple( ZZ );
Q
gap> Display( Q );
<An external ring residing in the CAS Maple>
gap> F2xyz := F2 * "x,y,z";
GF(2)[x,y,z]
gap> Display( F2xyz );
<An external ring residing in the CAS Maple>
gap> Qxyz := Q * "x,y,z";
Q[x,y,z]
gap> Display( Qxyz );
```

```
<An external ring residing in the CAS Maple>
gap> Dxyz := RingOfDerivations( Qxyz, "Dx,Dy,Dz" );
Q[x,y,z]<Dx,Dy,Dz>
gap> Display( Dxyz );
<An external ring residing in the CAS Maple>
gap> Exyz := ExteriorRing( Qxyz, "e,f,g" );
Q{e,f,g}
gap> Display( Exyz );
<An external ring residing in the CAS Maple>
#@fi
```

Chapter 2

Installation of the RingsForHomalg Package

To install this package just extract the package's archive file to the GAP `pkg` directory.

By default the RingsForHomalg package is not automatically loaded by GAP when it is installed. You must load the package with

```
LoadPackage( "RingsForHomalg" );
```

before its functions become available.

Please, send us an e-mail if you have any questions, remarks, suggestions, etc. concerning this package. Also, we would be pleased to hear about applications of this package.

The authors.

Chapter 3

The Ring Table

3.1 An Example for a Ring Table - Singular

todo: introductory text, mention: transposed matrices, the macros, refer to the philosophy

3.1.1 BasisOfRowModule (in the homalg table for Singular)

▷ `BasisOfRowModule(`*M*`)` (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro `BasisOfRowModule` (3.1.2) inside the computer algebra system.

Code

```
BasisOfRowModule :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      "unknown_number_of_rows",
      NrColumns( M ),
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = BasisOfRowModule(", M, ")" ],
      "need_command",
      "BasisOfModule"
    );

    return N;

  end,
```

3.1.2 BasisOfRowModule (Singular macro)

▷ `BasisOfRowModule(`*M*`)` (function)

Returns:

Code

```
    BasisOfRowModule := "\n\
proc BasisOfRowModule (matrix M)\n\
{\n\
  return(std(M));\n\
}\n\n",
```

3.1.3 BasisOfColumnModule (in the homalg table for Singular)

▷ `BasisOfColumnModule(`*M*`)` (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro `BasisOfColumnModule` (3.1.4) inside the computer algebra system.

Code

```
BasisOfColumnModule :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      NrRows( M ),
      "unknown_number_of_columns",
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = BasisOfColumnModule(", M, ")" ],
      "need_command",
      "BasisOfModule"
    );

    return N;

  end,
```

3.1.4 BasisOfColumnModule (Singular macro)

▷ `BasisOfColumnModule(`*M*`)` (function)

Returns:

Code

```
    BasisOfColumnModule := "\n\
proc BasisOfColumnModule (matrix M)\n\
{\n\
  return(Involution(BasisOfRowModule(Involution(M))));\n\
}\n\n",
```

3.1.5 DecideZeroRows (in the homalg table for Singular)

▷ `DecideZeroRows(`*A, B*`)` (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro DecideZeroRows (3.1.6) inside the computer algebra system. The rows of *B* must form a basis (see BasisOfRowModule (3.1.1)).

Code

```
DecideZeroRows :=
  function( A, B )
    local N;

    N := HomalgVoidMatrix(
      NrRows( A ),
      NrColumns( A ),
      HomalgRing( A )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = DecideZeroRows(", A, B, ")" ],
      "need_command",
      "DecideZero"
    );

    return N;

  end,
```

3.1.6 DecideZeroRows (Singular macro)

▷ DecideZeroRows(*A*, *B*) (function)

Returns:

Code

```
    DecideZeroRows := "\n\
proc DecideZeroRows (matrix A, module B)\n\
{\n\
  attrib(B,\"isSB\",1);\n\
  return(reduce(A,B));\n\
}\n\n",
```

3.1.7 DecideZeroColumns (in the homalg table for Singular)

▷ DecideZeroColumns(*A*, *B*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro DecideZeroColumns (3.1.8) inside the computer algebra system. The columns of *B* must form a basis (see BasisOfColumnModule (3.1.3)).

Code

```
DecideZeroColumns :=
  function( A, B )
    local N;

    N := HomalgVoidMatrix(
      NrRows( A ),
```

```
      NrColumns( A ),
      HomalgRing( A )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = DecideZeroColumns(", A, B, ")" ],
      "need_command",
      "DecideZero"
    );

    return N;

  end,
```

3.1.8 DecideZeroColumns (Singular macro)

▷ DecideZeroColumns(*A, B*) (function)

Returns:

Code

```
    DecideZeroColumns := "\n\
proc DecideZeroColumns (matrix A, matrix B)\n\
{\n\
  return(Involution(DecideZeroRows(Involution(A),Involution(B))));\n\
}\n\n",
```

3.1.9 SyzygiesGeneratorsOfRows (in the homalg table for Singular)

▷ SyzygiesGeneratorsOfRows(*M*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro SyzygiesGeneratorsOfRows (3.1.10) inside the computer algebra system.

Code

```
SyzygiesGeneratorsOfRows :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      "unknown_number_of_rows",
      NrRows( M ),
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = SyzygiesGeneratorsOfRows(", M, ")" ],
      "need_command",
      "SyzygiesGenerators"
    );

    return N;

  end,
```

3.1.10 SyzygiesGeneratorsOfRows (Singular macro)

▷ SyzygiesGeneratorsOfRows(*M*) (function)

Returns:

Code

```
    SyzygiesGeneratorsOfRows := "\n\
proc SyzygiesGeneratorsOfRows (matrix M)\n\
{\n\
  return(SyzForHomalg(M));\n\
}\n\n",
```

3.1.11 SyzygiesGeneratorsOfColumns (in the homalg table for Singular)

▷ SyzygiesGeneratorsOfColumns(*M*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro SyzygiesGeneratorsOfColumns (3.1.12) inside the computer algebra system.

Code

```
SyzygiesGeneratorsOfColumns :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      NrColumns( M ),
      "unknown_number_of_columns",
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = SyzygiesGeneratorsOfColumns(", M, ")" ],
      "need_command",
      "SyzygiesGenerators"
    );

    return N;

  end,
```

3.1.12 SyzygiesGeneratorsOfColumns (Singular macro)

▷ SyzygiesGeneratorsOfColumns(*M*) (function)

Returns:

Code

```
    SyzygiesGeneratorsOfColumns := "\n\
proc SyzygiesGeneratorsOfColumns (matrix M)\n\
{\n\
  return(Involution(SyzForHomalg(Involution(M))));\n\
}\n\n",
```

3.1.13 BasisOfRowsCoeff (in the homalg table for Singular)

▷ BasisOfRowsCoeff(*M, T*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro BasisOfRowsCoeff (3.1.14) inside the computer algebra system.

Code

```
BasisOfRowsCoeff :=
  function( M, T )
    local v, N;

    v := homalgStream( HomalgRing( M ) )!.variable_name;

    N := HomalgVoidMatrix(
      "unknown_number_of_rows",
      NrColumns( M ),
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, T, " = BasisOfRowsCoeff(", M, ")" ],
      "need_command",
      "BasisCoeff"
    );

    return N;

  end,
```

3.1.14 BasisOfRowsCoeff (Singular macro)

▷ BasisOfRowsCoeff(*M, T*) (function)

Returns:

Code

```
    BasisOfRowsCoeff := """
proc BasisOfRowsCoeff (matrix M)
{
  matrix B = BasisOfRowModule(M);
  option(noredSB);
  matrix T = lift(M,B);
  option(redSB);
  return(B,T);
}

 """,
```

3.1.15 BasisOfColumnsCoeff (in the homalg table for Singular)

▷ BasisOfColumnsCoeff(*M, T*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro BasisOfColumnsCoeff (3.1.16) inside the computer algebra system.

Code

```
BasisOfColumnsCoeff :=
  function( M, T )
    local v, N;

    v := homalgStream( HomalgRing( M ) )!.variable_name;

    N := HomalgVoidMatrix(
      NrRows( M ),
      "unknown_number_of_columns",
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, T, " = BasisOfColumnsCoeff(", M, ")" ],
      "need_command",
      "BasisCoeff"
    );

    return N;

  end,
```

3.1.16 BasisOfColumnsCoeff (Singular macro)

▷ BasisOfColumnsCoeff(*M, T*) (function)

Returns:

Code

```
    BasisOfColumnsCoeff := """
proc BasisOfColumnsCoeff (matrix M)
{
  matrix B,T = BasisOfRowsCoeff(Involution(M));
  return(Involution(B),Involution(T));
}

 """,
```

3.1.17 DecideZeroRowsEffectively (in the homalg table for Singular)

▷ DecideZeroRowsEffectively(*A, B, T*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro DecideZeroRowsEffectively (3.1.18) inside the computer algebra system. The rows of *B* must form a basis (see BasisOfRowModule (3.1.1)).

Code

```
DecideZeroRowsEffectively :=
  function( A, B, T )
    local v, N;
```

```
    v := homalgStream( HomalgRing( A ) )!.variable_name;

    N := HomalgVoidMatrix(
      NrRows( A ),
      NrColumns( A ),
      HomalgRing( A )
    );

    homalgSendBlocking(
      [ "matrix ", N, T, " = DecideZeroRowsEffectively(", A, B, ")" ],
      "need_command",
      "DecideZeroEffectively"
    );

    return N;

  end,
```

3.1.18 DecideZeroRowsEffectively (Singular macro)

▷ DecideZeroRowsEffectively(*A*, *B*, *T*) (function)

Returns:

Code

```
    DecideZeroRowsEffectively := """
proc DecideZeroRowsEffectively (matrix A, module B)
{
  attrib(B,"isSB",1);
  matrix M = reduce(A,B);
  matrix T = lift(B,M-A);
  return(M,T);
}

 """,
```

3.1.19 DecideZeroColumnsEffectively (in the homalg table for Singular)

▷ DecideZeroColumnsEffectively(*A*, *B*, *T*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro DecideZeroColumnsEffectively (3.1.20) inside the computer algebra system. The columns of *B* must form a basis (see BasisOfColumnModule (3.1.3)).

Code

```
DecideZeroColumnsEffectively :=
  function( A, B, T )
    local v, N;

    v := homalgStream( HomalgRing( A ) )!.variable_name;

    N := HomalgVoidMatrix(
      NrRows( A ),
      NrColumns( A ),
```

```
        HomalgRing( A )
      );

      homalgSendBlocking(
        [ "matrix ", N, T, " = DecideZeroColumnsEffectively(", A, B, ")" ],
        "need_command",
        "DecideZeroEffectively"
      );

      return N;

    end,
```

3.1.20 DecideZeroColumnsEffectively (Singular macro)

▷ DecideZeroColumnsEffectively(*A, B, T*) (function)

Returns:

Code

```
      DecideZeroColumnsEffectively := """
  proc DecideZeroColumnsEffectively (matrix A, matrix B)
  {
    matrix M,T = DecideZeroRowsEffectively(Involution(A),Involution(B));
    return(Involution(M),Involution(T));
  }

   """,
```

3.1.21 RelativeSyzygiesGeneratorsOfRows (in the homalg table for Singular)

▷ RelativeSyzygiesGeneratorsOfRows(*M, M2*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro RelativeSyzygiesGeneratorsOfRows (3.1.22) inside the computer algebra system.

Code

```
  RelativeSyzygiesGeneratorsOfRows :=
    function( M, M2 )
      local N;

      N := HomalgVoidMatrix(
        "unknown_number_of_rows",
        NrRows( M ),
        HomalgRing( M )
      );

      homalgSendBlocking(
        [ "matrix ", N, " = RelativeSyzygiesGeneratorsOfRows(", M, M2, ")" ],
        "need_command",
        "SyzygiesGenerators"
      );

      return N;
```

```
    end,
```

3.1.22 RelativeSyzygiesGeneratorsOfRows (Singular macro)

▷ RelativeSyzygiesGeneratorsOfRows(*M, M2*) (function)

Returns:

Code

```
    RelativeSyzygiesGeneratorsOfRows := "\n\
proc RelativeSyzygiesGeneratorsOfRows (matrix M1, matrix M2)\n\
{\n\
  return(BasisOfRowModule(modulo(M1, M2)));\n\
}\n\n",
```

3.1.23 RelativeSyzygiesGeneratorsOfColumns (in the homalg table for Singular)

▷ RelativeSyzygiesGeneratorsOfColumns(*M, M2*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro RelativeSyzygiesGeneratorsOfColumns (3.1.24) inside the computer algebra system.

Code

```
RelativeSyzygiesGeneratorsOfColumns :=
  function( M, M2 )
    local N;

    N := HomalgVoidMatrix(
      NrColumns( M ),
      "unknown_number_of_columns",
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = RelativeSyzygiesGeneratorsOfColumns(", M, M2, ")" ],
      "need_command",
      "SyzygiesGenerators"
    );

    return N;

  end,
```

3.1.24 RelativeSyzygiesGeneratorsOfColumns (Singular macro)

▷ RelativeSyzygiesGeneratorsOfColumns(*M, M2*) (function)

Returns:

Code

```
    RelativeSyzygiesGeneratorsOfColumns := "\n\
proc RelativeSyzygiesGeneratorsOfColumns (matrix M1, matrix M2)\n\
{\n\
```

```
    return(Involution(RelativeSyzygiesGeneratorsOfRows(Involution(M1),Involution(M2))));\n\
  }\n\n",
```

3.1.25 ReducedSyzygiesGeneratorsOfRows (in the homalg table for Singular)

▷ ReducedSyzygiesGeneratorsOfRows(*M*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro ReducedSyzygiesGeneratorsOfRows (3.1.26) inside the computer algebra system.

Code

```
ReducedSyzygiesGeneratorsOfRows :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      "unknown_number_of_rows",
      NrRows( M ),
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = ReducedSyzygiesGeneratorsOfRows(", M, ")" ],
      "need_command",
      "SyzygiesGenerators"
    );

    return N;

  end,
```

3.1.26 ReducedSyzygiesGeneratorsOfRows (Singular macro)

▷ ReducedSyzygiesGeneratorsOfRows(*M*) (function)

Returns:

Code

```
    ReducedSyzForHomalg := "\n\
proc ReducedSyzForHomalg (matrix M)\n\
{\n\
  return(matrix(nres(M,2)[2]));\n\
}\n\n",
    ReducedSyzygiesGeneratorsOfRows := "\n\
proc ReducedSyzygiesGeneratorsOfRows (matrix M)\n\
{\n\
  return(ReducedSyzForHomalg(M));\n\
}\n\n",
```

3.1.27 ReducedSyzygiesGeneratorsOfColumns (in the homalg table for Singular)

▷ ReducedSyzygiesGeneratorsOfColumns(*M*) (function)

Returns:

This is the entry of the homalg table, which calls the corresponding macro ReducedSyzygiesGeneratorsOfColumns (3.1.28) inside the computer algebra system.

Code

```
ReducedSyzygiesGeneratorsOfColumns :=
  function( M )
    local N;

    N := HomalgVoidMatrix(
      NrColumns( M ),
      "unknown_number_of_columns",
      HomalgRing( M )
    );

    homalgSendBlocking(
      [ "matrix ", N, " = ReducedSyzygiesGeneratorsOfColumns(", M, ")" ],
      "need_command",
      "SyzygiesGenerators"
    );

    return N;

  end,
```

3.1.28 ReducedSyzygiesGeneratorsOfColumns (Singular macro)

▷ ReducedSyzygiesGeneratorsOfColumns(*M*) (function)

Returns:

Code

```
    ReducedSyzygiesGeneratorsOfColumns := "\n\
proc ReducedSyzygiesGeneratorsOfColumns (matrix M)\n\
{\n\
  return(Involution(ReducedSyzForHomalg(Involution(M))));\n\
}\n\n",
```

References

[hom22] homalg project authors. The homalg project – Algorithmic Homological Algebra. (https://homalg-project.github.io/prj/homalg_project), 2003–2022. 4

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